INTUITIONISTIC FUZZY NANO SEMI GENERALIZED CLOSED SETS

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Abstract. In this article, we investigate intuitionistic fuzzy nano semi generalized closed sets in terms of intuitionistic fuzzy interior and closure operators. Also we study some of its results.

Keywords: IFNs-g interior, IFNs-g closure and IFNsg-closed set.
MSC (2010): Primary: 54B05; Secondary: 08A72

1. Introduction

Levine [4] gave the idea of generalizing closed sets in topological spaces by comparing the closure of a subset with its open supersets. In 1987, P.Bhattacharyya, et. al [6] introduced the notion of semi generalized closed sets in topological spaces. Further many researchers extended the study of generalized closed sets on the basis of generalized open sets. Lellis Thivagar, et.al[7][13] defined Nano topology in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure. A. Stephan Antony Raj, et.al [3] introduced intuitionistic fuzzy nano topological space (IFNTS) with respect to a subset *X* of an universe *U* and studied its application. Further the present authors [5] studied the properties of intuitionistic fuzzy closed sets and its generalization.

In this article, we discuss the concept of intuitionistic fuzzy nano semi generalized closed sets in intuitionistic fuzzy nano topological spaces and its properties are also introduced and investigated.

2. Preliminaries

Definition 1[3]. Let U be the universe, R be an intuitionistic fuzzy equivalence relation on U and $\tau_R(X) = \{1\sim, 0\sim, IFL_R(X), IFU_R(X), IFB_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) $1\sim$ and $0\sim \in \tau_R(X)$.

(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the intuitionistic fuzzy nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the intuitionistic fuzzy nano topological space. The elements of $\tau_R(X)$ are called as intuitionistic fuzzy nano-open sets. If $(U, \tau_R(X))$ is a intuitionistic fuzzy nano topological space(In short IFNTS) where $X \subseteq U$ and if $A \subseteq U$, then the intuitionistic fuzzy nano interior of A is defined as the union of all intuitionistic fuzzy nano-open subsets of A and it is denoted by $IFNInt(A)$. $IFNInt(A)$ is the largest intuitionistic fuzzy nano-open subset of A. The intuitionistic fuzzy nano closure of A is defined as the intersection of all intuitionistic fuzzy nano closed sets(In short IFNCS) containing A and it is denoted by $IFNCl(A)$. That is, $IFNCl(A)$ is the smallest intuitionistic fuzzy nano closed set containing A.

Definition 2[3]. Let (U, τ_R(X)) be a intuitionistic fuzzy nano topological space and A$\subseteq$U. Then A is said to be

a) intuitionistic fuzzy nano semi-open if A$\subseteq$IFNCl(IFNInt(A)).

b) intuitionistic fuzzy nano pre-open if A$\subseteq$IFNInt(IFNCl(A)).

c) intuitionistic fuzzy nano α-open if A ⊆ IFNInt(IFNCl(IFNInt(A))).

IFNSO(U, X), IFNPO(U, X) and τ_R (X) respectively denote the families of all intuitionistic fuzzy nano semi-open, intuitionistic fuzzy nano pre-open and intuitionistic fuzzy nano α-open subsets of U.

Definition 3[3]. Let (U, τ_R (X)) be a intuitionistic fuzzy nano topological space and A ⊆ U. A is said to be intuitionistic fuzzy nano α-closed (resp., intuitionistic fuzzy nano semi-closed(In short IFNSC), intuitionistic fuzzy nano pre-closed), if its complement is nano α-open (resp., intuitionistic fuzzy nano semi-open, intuitionistic fuzzy nano pre-open).

Definition 4[5]. Let (U, τ_R(X)) be an intuitionistic fuzzy nano topological space. A subset A of (U, τ_R(X)) is called intuitionistic fuzzy nano generalized closed set (briefly IFNg- closed) if IFNCl(A) ⊆ V where A ⊆ V and V is intuitionistic fuzzy nano open(In short IFNOS).

Definition 5[7]. A subset A of a nano topological space U is said to be nano semi-generalized closed (briefly, nano sg-closed), if N scl(A) ⊆ G whenever G is nano semi-open and A ⊆ G. The set A is said to be nano semi-generalized open (nano sg-open) if A^C is nano sg-closed.

Definition 6[13]. A subset A of a nano topological space U is said to be nano semi-generalized closed (briefly, nano sg-closed), if N scl(A) ⊆ G whenever G is nano semi-open and A ⊆ G. The set A is said to be nano semi-generalized open (nano sg-open) if A^C is nano sg-closed.

Definition 7[13]. If A ⊆ U, then the nano semi - generalized closure denoted by N sgcl(A) is defined as the smallest nano sg-closed set containing A. The nano semi-generalized interior of A, denoted by N sgint(A) is defined as the largest nano sg-open subset of A.

3. Intuitionistic Fuzzy nano semi generalized closed sets

In this section, we extend the concept of intuitionistic fuzzy nano generalized closed sets to intuitionistic fuzzy nano semi generalized closed sets that are independent of intuitionistic fuzzy nano generalized closed sets and defined by comparing the intuitionistic fuzzy nano semi closure of a set with its intuitionistic fuzzy nano semi-open supersets.

Throughout this paper $(T, \tau_R(X))$ is an intuitionistic fuzzy nano topological space(In short IFNTS) with respect to X where $X \subseteq T$, R is an equivalence relation on T, T/R denotes the family of equivalence classes of T by R.

Definition 8. A subset A of a intuitionistic fuzzy nano topological space T is said to be intuitionistic fuzzy nano semi-generalized closed (briefly, IFNsg-closed), if IFNscl(A) ⊆ G whenever G is intuitionistic fuzzy nano semi-open(In short IFNSO) and A ⊆ G. The set A is said to be intuitionistic fuzzy nano semi-generalized open (IFNsg-open) if A^C is intuitionistic fuzzy nano semi-generalized closed(IFNsg-closed).

Example 1. Let (T,R) be an intuitionistic fuzzy approximation space(In short IFAS) where T = {x_1, x_2, x_3} with R = {⟨(x_1, x_1),1,0⟩, ⟨(x_1, x_2),0.50,0.50⟩, ⟨(x_2, x_1),0.50,0.50⟩, ⟨(x_2, x_2),1,0⟩, ⟨(x_2, x_3),0.30,0.70⟩, ⟨(x_3, x_2),0.30,0.70⟩, ⟨(x_3, x_3),1,0⟩, ⟨(x_1, x_3),0.40,0.60⟩, ⟨(x_3, x_1),0.40,0.60⟩}.

Let B = {⟨ x_1,0.50,0.50⟩, ⟨ x_2,0.50,0.50⟩, ⟨ x_3,0.30,0.60⟩} be an intuitionistic fuzzy set(In short IFS)on T then

$\tau_R(X)$={1∼,0∼,{⟨x_1,0.70,0.30⟩,⟨x_2,0.50,0.50⟩,⟨x_3,0.60,0.30⟩},{⟨x_1,0.50,0.50⟩,⟨x_2,0.50,0.50⟩,⟨x_3,0.60,0.30⟩},{⟨x_1,0.50,0.50⟩, ⟨x_2,0.50,0.50⟩,⟨ x_3,0.30,0.60⟩}}.

Here, C = {⟨ x_1,0.50,0.50⟩, ⟨ x_2,0.50,0.50⟩, ⟨ x_3,0.30,0.60⟩}is an IFNSO set, then IFNscl(B) ⊆ C and B ⊆ C. Therefore, B is IFNsg-closed.

Definition 9. If A ⊆ T, then the intuitionistic fuzzy nano semi - generalized closure denoted by IFNs-gcl(A) is defined as the smallest intuitionistic fuzzy nano sg-closed set containing A. The intuitionistic fuzzy nano semi-generalized interior of A , denoted by IFNs-gint(A) is defined as the largest intuitionistic fuzzy nano sg-open subset of A.

Theorem 1. A set A is IFNsg-closed in T if and only if IFN scl(A) –A has no non-empty, IFNSC.

Proof. Let A be IFNsg-closed and F be a IFNSC subset of IFNscl(A) − A. Then $(\text{IFN scl}(A) - A)^C \subseteq F^C$ and F^C is IFNSO. That is, $(\text{IFN scl}(A) \cap A^C)^C \subseteq F^C$. Therefore, $A \cup (\text{IFN sint}(A^C)) \subseteq F^C$. Thus F^C is IFNSO and $A \subseteq F^C$. Since A is

IFNsg-closed, IFNscl(A) ⊆ F^C. That is, F ⊆ $(IFNscl(A))^C$. Thus, F ⊆ (IFNscl(A)) ∩ $(IFNscl(A))^C$ = 0∼. Therefore, F = 0∼. Conversely, let IFNscl(A) − A have no non-empty, IFNSC. Let G be IFNSO in T such that A ⊆ G. If IFNscl(A) * G, then IFNscl(A) ∩ G^C = 0∼. And, IFNscl(A) ∩ G^C ⊆ IFNscl(A) − A, since A ⊆ G. Thus IFNscl(A) ∩ G^C is a non-empty IFNSC of IFNscl(A) − A, which is a contradiction. Therefore, IFNscl(A) ⊆ G whenever G is IFNSO and A ⊆ G. That is, A is IFNsg-closed in T.

The following theorem provides a condition under which a IFNsg-closed is IFNSC.

Theorem 2. Let A be IFNsg-closed. Then A is IFNSC if and only if IFNscl(A) −A is IFNSC.

Proof. Let A be IFNsg-closed. If A is IFNSC, IFNscl(A) = A and hence IFNscl(A) −A = 0∼ which is IFNSC. Conversely let IFNscl(A) −A be IFNSC. Then IFNscl(A) −A is IFNsg-closed. Then IFN scl(A) − A does not contain any non-empty, IFNSC. Therefore, IFNscl(A) − A = 0∼. That is, IFNscl(A) = A. Therefore, A is IFNSC.

Now, we derive the forms of IFNsg-closed for various cases of approximations.

Theorem 3. If $IFL_R(X) = IFU_R(X)$ in a IFNTS T, then any A ⊆ $[IFL_R(X)]^C$ and $[IFL_R(X)]^C$ ∪B where B ⊆ $IFL_R(X)$ are the only IFNsg-closed sets in T.

Proof. When IFLR (X) = $IFU_R(X)$, $\tau_R(X)$ = {1∼,0∼, $IFL_R(X)$}. Also, 0∼ and any A ⊇ $IFL_R(X)$ are the only IFNSO sets in T. If A ⊆ $IFL_R(X)$, then IFNscl(A) = T and the IFNSO sets containing A are those sets B for which $IFL_R(X)$ ⊆ B. Thus, IFNscl(A) ⊆ G, not for every IFNSO such that A ⊆ G. Therefore, A is not IFNsg-closed. If A ⊆ $[IFL_R(X)]^C$, then Nscl(A) = A, since any subset of $[IFL_R(X)]^C$ is IFNSC in T. Thus, IFN scl(A) = A ⊆ G whenever G is IFNSO and A ⊆ G. Therefore, A is IFNsg-closed. If A ⊇$IFL_R(X)$ and A = T, IFN cl(A) = T and the IFNSO containing A are A and T. Therefore, IFNscl(A) * A. Therefore, any A ⊇ $IFL_R(X)$ and A = T is not IFNsg-closed. If A ⊇ $[IFL_R(X)]^C$,then IFNscl(A) ⊆ G whenever G is IFNSO and G ⊆ A, since T is the only IFNSO set containing A. Therefore, if A ⊇ $[IFL_R(X)]^C$, then A is IFNsg-closed. When A has atleast one element of $IFL_R(X)$ and exactly one element, say y, of $[IFL_R(X)]^C$ where $IFL_R(X)$ is not a singleton set, IFNscl(A) = T. But $IFL_R(X)$ ∪ {y} is a nano semi-open set containing A and IFNscl(A) = T * $IFL_R(X)$∪{y}. Therefore, A is not IFNsg-closed.

Thus, the only IFNsg-closed sets in T are subsets of $[IFL_R(X)]^C$ and any $A \supset [IFL_R(X)]^C$.

Theorem 4. If $IFL_R(X) = 0\sim$ and $IFU_R(X) = 1\sim$, then the only IFNsg-closed sets in T are subsets of $[IFU_R(X)]^C$ and any $A \supset [IFU_R(X)]^C$.

Proof. If $IFL_R(X) = 0\sim$ and $IFU_R(X) = 1\sim$, then $\tau_R(X) = \{1\sim, 0\sim, IFU_R(X)\}$. Also, $0\sim$ and those sets A for which $A \supseteq IFU_R(X)$ are the only IFNSO sets in T. Therefore, the sets A for which $A \subseteq [IFU_R(X)]^C$ are the only IFNSC sets in T. If $A \subseteq IFU_R(X)$, then IFNscl(A) = T. But $IFU_R(X)$ is a IFNSO set containing A, for which IFNscl(A) $*$ G. Therefore, A is not IFNsg-closed. If $A \supset IFU_R(X)$ and A = T, then IFNscl(A) = T. But, for G = A, which is a IFNSO set containing itself, IFN scl(A) $*$ G. Therefore, A is not IFNsg-closed. If $A \subseteq [IFU_R(X)]^C$, then IFNscl(A) = A and hence for every IFNSO set G such that $A \subseteq G$, IFNscl(A) $\subseteq$ G. Therefore A is IFNsg-closed. If $A \supset [IFU_R(X)]^C$, then T is the only IFNSO set containing A and hence IFNscl(A) $\subseteq$ G whenever G is IFNSO and $A \subseteq G$. Therefore, A is IFNsg-closed. If A has one element, say x of $IFU_R(X)$ and atleast one element of $[IFU_R(X)]^C$, then IFNscl(A) = T. Since any set containing $IFU_R(X)$ is IFNSO in T, $A \cup (IFU_R(X))$ and any set containing $A \cup (IFU_R(X))$ are IFNSO sets containing A. But, IFNscl(A) = T $*$ $A \cup (IFU_R(X))$.Therefore, A is not IFNsg-closed in T. Thus, only subsets of $[IFU_R(X)]^C$ and any $A \supset [IFU_R(X)]^C$ are IFNsg-closed in T, when $IFL_R(X) = 0\sim$ and $IFU_R(X) = 1\sim$.

Theorem 5. If $IFU_R(X) = 1\sim$ and $IFL_R(X) = 0\sim$ in a IFNTS T, then every subset of T is IFNsg-closed.

Proof. $1\sim$, $0\sim$, $IFL_R(X)$ and $IFB_R(X)$ are the only sets in T which are IFNO, IFNSO and IFNSC in T. If $A \subseteq IFL_R(X)$, then $IFL_R(X)$ and T are the only IFNSO sets containing A and IFNscl(A) = $IFL_R(X)$. Therefore, IFNscl(A) $\subseteq$ G whenever G is IFNSO and $A \subseteq G$. Thus, A is IFNsg-closed. If $A \subseteq IFB_R(X)$, then $IFB_R(X)$ and T are the only IFNSO sets containing A and IFNscl(A) = $IFB_R(X)$. Therefore, A is IFNsg-closed. If $A \supset IFL_R(X)$ or $A \supset IFB_R(X)$, then T is the only IFNSO set containing A and hence A is IFNsg-closed. If A contains atleast one element of $IFL_R(X)$ and atleast one element of $IFB_R(X)$, then T is the only IFNSO set containing A. Therefore, A is IFNsg-closed. Thus, every subset of T is IFNsg-closed, if $IFU_R(X) = 1\sim$ and $IFL_R(X) = 0\sim$.

Theorem 6. If $IFL_R(X) = IFU_R(X)$ where $IFL_R(X) = 0\sim$ and $IFU_R(X) = 1\sim$, then any subset of $[IFL_R(X)]^C$, any subset of $[IFB_R(X)]^C$ and those subsets of T which have atleast one element each of $IFL_R(X)$, $IFB_R(X)$ and $[IFU_R(X)]^C$ are the only IFNsg-closed sets in T.

Proof. $\tau_R(X) = \{1\sim, 0\sim, IFL_R(X), IFB_R(X), IFU_R(X)\}$. Also $1\sim, 0\sim, IFL_R(X), IFB_R(X)$, $A \supseteq IFU_R(X)$, $IFL_R(X) \cup [IFU_R(X)]^C$, $IFB_R(X) \cup [IFU_R(X)]^C$, $A \cup IFL_R(X)$ and $A \cup IFB_R(X)$, where $A \subset [IFU_R(X)]^C$ are the only IFNSO sets in T. Therefore, the IFNSC sets in T are $1\sim, 0\sim, [IFL_R(X)]^C, [IFB_R(X)]^C$, $A \subseteq [IFU_R(X)]^C$, $IFB_R(X)$, $IFL_R(X)$, $(A \cup IFB_R(X))^C$ and $(A \cup IFL_R(X))^C$ where $A \subset [IFU_R(X)]^C$.

Let $A \subseteq IFL_R(X)$. Then $IFNscl(A) = IFL_R(X)$. Also $IFL_R(X)$, $A \supseteq IFU_R(X)$, $IFL_R(X) \cup [IFU_R(X)]^C$ and $A \cup IFL_R(X)$ where $A \subset [IFU_R(X)]^C$ are the IFNSO sets containing A. Therefore, $IFNscl(A) \subseteq G$ whenever G is IFNSO and $A \subseteq G$. Thus, A is IFNsg-closed.If $A \subseteq IFB_R(X)$, then $IFNscl(A) = IFB_R(X)$ and $IFB_R(X)$, $A \supseteq IFU_R(X)$, $IFB_R(X) \cup [IFU_R(X)]^C$ and $A \cup IFB_R(X)$ where $A \subset [IFU_R(X)]^C$ are the IFNSO sets containing A. Therefore, $IFNscl(A) \subseteq G$ whenever G is IFNSO and $A \subseteq G$. Thus, A is IFNsg-closed.

If $A = IFL_R(X) \cup [IFU_R(X)]^C$, then $IFNscl(A) = IFL_R(X) \cup [IFU_R(X)]^C = A$. And $IFL_R(X) \cup [IFU_R(X)]^C$ and T are the only IFNSO sets containing A. Therefore, $IFNscl(A) \subseteq G$ whenever G is IFNSO and $A \subseteq G$ and hence A is IFNsg-closed.

If $A = IFB_R(X) \cup [IFU_R(X)]^C$, then the IFNSO sets containing A are A and T. Also $IFNscl(A) = A$. Therefore, $IFNscl(A) \subseteq G$ whenever G is IFNSO and $A \subseteq G$. Thus, A is IFNsg-closed.

If $A = [IFU_R(X)]^C$, then the IFNSO sets containing A are T, $IFL_R(X) \cup A$ and $IFB_R(X) \cup A$ and $IFNscl(A) = A$. Thus, $IFNscl(A) \subseteq G$ whenever G is IFNSO and $A \subseteq G$. Thus, A is IFNsg-closed.

Thus, combining all the above cases together, we see that subsets of $[IFB_R(X)]^C$

and subsets of $[IFL_R(X)]^C$ are IFN sg-closed in T.

If A has atleast one element each of $IFL_R(X)$, $IFB_R(X)$ and $[IFU_R(X)]^C$, then IFN scl(A) = T and T is the only IFNSO set containing A. Therefore, A is IFNsg-closed. If A has atleast one element of $IFL_R(X)$ and atleast one element of $IFB_R(X)$, but no element of $[IFU_R(X)]^C$, then $IFU_R(X)$ and T are the IFNSO sets containing A and IFN scl(A) = T. Therefore, IFN scl(A) * $IFU_R(X)$, a IFNSO set containing A. Thus, A is not IFN sg-closed.

Thus, the only IFNsg-closed sets in T are subsets of $[IFL_R(X)]^C$, subsets of $[IFB_R(X)]^C$ and those sets which have atleast one element each of $IFL_R(X)$, $IFB_R(X)$ and $[IFU_R(X)]^C$.

Theorem 7. If $IFL_R(X) = IFU_R(X)$ in T, then $A \cup B$ is IFN sg-closed whenever A and B are IFN sg-closed.

Proof. The only IFNsg-closed sets in T are subsets of $[IFL_R(X)]^C$ and

$IF[L_R(X)]^C \cup B$ where $B \subseteq IF\,L_R(X)$. Let A and B be IFN sg-closed in T.

Case 1: Let $A \subseteq [IFL_R(X)]^C$ and $B \subseteq [IFL_R(X)]^C$. Then $A \cup B \subseteq [IFL_R(X)]^C$ which is IFN sg-closed.

Case 2: Let $A \subseteq [IFL_R(X)]^C$ and $B = [IFL_R(X)]^C \cup Y$ where $Y \subseteq IFL_R(X)$. Then $A \cup B \subseteq [IFL_R(X)]^C \cup Y = B$ which is IFN sg-closed.

Case 3: Let $A = [IFL_R(X)]^C \cup Y$ and $B = [IFL_R(X)]^C \cup Z$ where Y and $Z \subseteq IFL_R(X)$. Then $A \cup B = [IFL_R(X)]^C \cup [Y \cup Z]$ where $Y \cup Z \subseteq IFL_R(X)$ and hence $A \cup B$ is IFN sg-closed.

Theorem 8. If $IFL_R(X) = 0_\sim$ and $IFUR(X) = 1_\sim$, then the union of IFN sg-closed sets is IFN sg-closed.

Proof. Since subsets of $[IFU_R(X)]^C$ and any set $\supset [IFU_R(X)]^C$ are nano sg-closed, if A and B are IFN sg-closed in T, then A and B are either subsets of $[IFU_R(X)]^C$ or contain $[IFU_R(X)]^C$. Then $A \cup B$ is also either a subset of $[IFU_R(X)]^C$ or contains $[IFU_R(X)]^C$ and hence is IFN sg-closed.

Theorem 9. The union of two IFN sg-closed sets is IFN sg-closed, if $U_R(X) = 1_\sim$ and $L_R(X) = 0_\sim$.

Proof. The result is obvious, since every subset of T is IFN sg-closed.

Theorem 10. If A and B are IFN sg-closed, then A∩B is IFN sg-closed in T.

Proof.

Case 1: Let $IFL_R(X) = IFU_R(X)$. Then any subset of $[IFL_R(X)]^C$ and any $A \supseteq [IFL_R(X)]^C$ are the only IFN sg-closed sets in T. If A and B are IFN sg-closed and $A, B \subseteq [IFL_R(X)]^C$ then $A \cap B \subseteq [IFL_R(X)]^C$ and hence is IFN sg-closed. If $A \supseteq [IFL_R(X)]^C$ and $B \supseteq [IFL_R(X)]^C$, then $A \cap B \supseteq [IFL_R(X)]^C$ and hence is IFN sg-closed. If $A \subseteq [IFL_R(X)]^C$ and $B \supseteq [IFL_R(X)]^C$, then $A \cap B \subseteq [IFL_R(X)]^C$ and hence is IFN sg-closed. Thus, if $IFL_R(X) = IFU_R(X)$, then $A \cap B$ is IFN sg-closed, whenever A and B are IFN sg-closed.

Case 2: Let $IFL_R(X) = 0_\sim$ and $IFU_R(X) = 1_\sim$. Then any subset of $[IFU_R(X)]^C$ and any $A \supseteq [IFU_R(X)]^C$ are the only IFN sg-closed sets in T. If A and B are IFN sg-closed then A and B are either subsets of $[IFU_R(X)]^C$ or $\supseteq [IFU_R(X)]^C$. Therefore, as in case 1, for different choices of A and B, it can be shown that $A \cap B$ is IFN sg-closed.

Case 3: Let $IFL_R(X) = 0_\sim$ and $IFU_R(X) = 1_\sim$. Since every subset of T is IFN sg-closed, intersection of IFN sg-closed sets is IFN sg-closed.

Case 4: Let $IFL_R(X) = IFU_R(X)$ where $IFL_R(X) = 0_\sim$ and $U_R(X) = 1_\sim$. Then subsets of $[IFL_R(X)]^C$, subsets of $[IFB_R(X)]^C$ and those sets which have atleast one element each of $IFL_R(X)$, $IFB_R(X)$ and $[IFU_R(X)]^C$ are the only IFN sg-closed sets in T. Let A and B be IFN sg-closed in T. If both A and B are subsets of $[IFL_R(X)]^C$ or subsets of $IFFB_R(X)]^C$ or both have atleast one element each of $IFL_R(X)$, $IFB_R(X)$ and $[IFU_R(X)]^C$, then $A \cap B$ is IFN sg-closed. If $A \subseteq [IFB_R(X)]^C$ and $B \subseteq [IFL_R(X)]^C$, then $A \cap B \subseteq [IFB_R(X)]^C$ and $[IFL_R(X)]^C$ and hence is IFN sg-closed. If $A \subseteq [IFL_R(X)]^C$ and if B has atleast one element each of

$IFL_R(X)$, $IFB_R(X)$ and $[IFU_R(X)]^C$, then $A \cap B \subsetneq [IFL_R(X)]^C$ and hence is IFN sg-closed. Similarly, if $A \subsetneq [IFB_R(X)]^C$ and if B has atleast one element each of $IFL_R(X)$, $IFB_R(X)$ and $[IFU_R(X)]^C$, then it can be shown that $A \cap B$ is IFN sg-closed

Remark 1. If $L_R(X) = 0\sim$ and $U_R(X) = 1\sim$ in a IFNTS, then those sets A for which $A \ast IFU_R(X)$ are the only IFN g-closed sets.

Proof. $\tau_R(X) = \{1\sim, 0\sim, U_R(X)\}$. If $A \subseteq IFU_R(X)$, then T and $IFU_R(X)$ are the IFNO sets containing A. Also, IFNcl(A) = 1~ and hence IFNcl(A) $\ast$ $U_R(X)$. Thus, N cl(A) $\ast$ G when $G = U_R(X)$. Therefore, A is not IFN g-closed. But, if $A \ast IFU_R(X)$, then T is the only IFNO set $\supseteq$ A and hence IFNcl(A) $\subseteq$ G whenever G is IFNO set and $G \supseteq A$. Therefore, A is IFN g-closed. Thus, only those subsets A of T such that $A \ast IFU_R(X)$ are IFN g-closed in T, if $IFL_R(X) = 0\sim$ and $IFU_R(X) = 1\sim$.

Theorem 11. If $IFL_R(X) = 0\sim$and $IFU_R(X) = 1\sim$, then any IFN sg-closed set is IFN g-closed.

Proof. The only IFN sg-closed sets are subsets of $[IFU_R(X)]^C$ and any set $\supset [IFU_R(X)]^C$. If $A \subsetneq [IFU_R(X)]^C$, then $A \ast IFU_R(X)$. If $A \supset [IFU_R(X)]^C$, then there exists atleast one element of A which is not in $IFU_R(X)$. That is, $A \ast IFU_R(X)$. Thus, in both cases, $A \ast IFU_R(X)$ and such sets are IFN g-closed in T by remark 1. Thus any IFN sg-closed set is IFN g-closed.

Theorem 12. If $IFL_R(X) = IFU_R(X)$ where $IFL_R(X) = 0\sim$ and $IFU_R(X) = 1\sim$, then every IFN g-closed set is IFN sg-closed.

Proof. Every $A \ast IFU_R(X)$ is IFN g-closed. When $A \ast IFU_R(X)$, A has atleast one element of $[IFU_R(X)]^C$ and A may or may not have an element of $IFL_R(X)$ or $IFB_R(X)$. If A has atleast one element of $IFL_R(X)$ but no element of $IFB_R(X)$, then $A \subsetneq IFL_R(X) \cup [IFU_R(X)]^C$. Therefore $A \subsetneq [IFB_R(X)]^C$ which is IFN sg-closed. If A has atleast one element of $IFB_R(X)$ but no element of $IFL_R(X)$, then $A \subsetneq$

$[IFL_R(X)]^C$ which is IFN sg-closed. If A has atleast one element of $IFL_R(X)$ and atleast one element of $IFB_R(X)$, then again A is IFN sg-closed. If A has only elements of $[IFU_R(X)]^C$, then A is IFN sg-closed. Thus any IFN g-closed is IFN sg-closed, if $IFL_R(X) = IFU_R(X)$.

Theorem 13. If $IFL_R(X) = IFU_R(X)$, then any IFN sg-closed set is IFN g-closed.

Proof. Any set $A * IFL_R(X)$ is IFN sg-closed. And a subset A of T is IFN sg-closed only if $A = [IFL_R(X)]^C$ or $[IFL_R(X)]^C \cup B$ where $B \subseteq IFL_R(X)$ and in both cases, $A * IFL_R(X)$. Therefore, any IFN sg-closed set is IFN g-closed.

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