Soret and Dufour Effects on Free Convection Heat and Mass Transfer from Vertical Surface in a Porous Medium with Viscous Dissipation, Heat Generation and Radiation

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Abstract

Numerical computation has been carried out for the study of non-linear coupled partial differential equations for combined effects of Soret and Dufour on free convection heat and mass transfer from vertical surface in a porous medium with viscous dissipation, heat generation and radiation. The governing differential equations of the problem have been transformed into a system of non- dimensional differential equations, which are then solved numerically using a Runge-Kutta Gill method along with shooting technique. The dimensionless velocity, temperature and concentration distributions are discussed numerically and presented through graphs. The numerical values of skin-friction coefficient and Nusselt number at the plate are derived.

Key words: Free convection; Porous medium; Viscous dissipation; Soret effect; Dufour effect; Thermal radiation.

Nomenclatures

S_r – Soret number

D_f – Dufour number

T- Temperature

x, y – Cartesian Coordinates along and normal to the surface, respectively

α_m- Thermal diffusivity

C- Concentration

C_p – Specific heat at constant pressure

C_s – Concentration susceptibility

Ra_x – Local Rayleigh number

u,v – Darcian velocities in the x and y directions

θ - Dimensionless temperature

ρ - density of the fluid

Ψ - Stream function

β_T – Coefficient of thermal expansion

β_c – Coefficient of concentration expansion

ϕ - Dimensionless concentration

η - Similarity variable

υ - Kinematic viscosity

D_m – Mass diffusivity

f – Dimensionless stream function

K – Darcy permeability

K_T – Thermal diffusion ratio

α^* - Mean absorption coefficient

R- Thermal radiation parameter

Ec – Eckert number

S – Heat source

Le- Lewis number

N- Sustentation parameter

Introduction

The thermal-diffusion and diffusion-thermo effects are interesting macroscopically physical phenomenon in fluid mechanics. Several studies have been reported, dealing with natural convection due to thermal buoyancy forces, mainly because of its importance in industrial and technological applications, such as geothermal energy, fibrous insulation, etc. However, less attention has been dedicated to so called double diffusive problems, where density gradients occur due to the effects of combined temperature and compositional buoyancy. It was observed that an energy flux can be resulted in not only by the temperature gradient but also by the concentration gradient. The heat transfer caused by concentration gradient is called the diffusion-thermo or Dufour effect. On the other hand, mass transfer caused by temperature gradients is called Soret or thermal diffusion effect. Thus Soret effect is referred to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient and the Dufour effect referred to the heat flux produced by a concentration gradient. Usually, in heat and mass transfer problems the variation of density with temperature and concentration give rise to a combined buoyancy force under natural convection. The heat and mass transfer simultaneously affect each other that create cross-diffusion effect. Soret and Dufour effects have been found to appreciably influence the flow field in free convection boundary layer over a vertical surface embedded in a porous medium. Steady and transient free convection in a fluid saturated porous medium has attracted considerable attention in the last years, due to many important engineering and geophysical applications. Recent books by [1,2,3] present a comprehensive account of the available information in the field. Free convective fluctuating magnetohydrodynamic flow through porous media past a vertical porous plate with variable temperature and heat source analyzed by [4].The hydromagnetic flow of a viscous incompressible fluid past an oscillating vertical plate embedded in a porous medium, radiation with viscous dissipation and variable heat and mass diffusion studied by [5]. Free convective flow of visco-elastic fluid in a vertical channel with dufour effect studied by [6]. Soret and Dufour effects on MHD free convective flow over a permeable stretching surface with chemical reaction and heat sink studied by [7]. Heat and mass transfer by natural convection from a vertical surface embedded in a fluid saturated darcian porous medium including soret, dufour effects with viscous dissipation effect in the presence of thermal radiation analyzed by [8]. Heat and mass transfer by natural convection from a vertical surface embedded in a fluid saturated darcian porous medium including soret and dufour effects with viscous dissipation effect studied by [9]. Chemical reaction effect on magnetohydrodynamic free convective surface over a moving vertical plane through porous medium discussed by [10]. C H Chen [11] studied MHD mixed convection of a power-law fluid past a stretching surface in the presence of thermal radiation and internal heat generation/absorption Dakshinamoorthy etal [12] analyzed the effects on both momentum and heat transfer problem with viscous dissipation and Joule heat transfer for an electrically conducting fluid past a continuously moving plate in the presence of a uniform transverse magnetic field. P. Geetha [13] studied the effects of Soret and Dufour on a steady free convection and mass transfer flow past a semi infinite flat plate with viscous dissipation in a porous medium. P.Geetha [14] investigated the Dufour and Soret effects on steady two-dimensional free convection heat and mass transfer along a horizontal plate in a porous medium.

The aim of this paper is to analyze the soret and dufour effects on free convection heat and mass transfer from vertical surface in a porous medium with viscous dissipation, heat generation and radiation.

Mathematical Formulation

The steady two dimensional natural convection in a porous medium saturated with a Newtonian fluid on a vertical flat plate is considered. The x-coordinate is measured along the surface and the y-coordinate normal to it shown in Fig. 1. The temperature of the ambient medium is T_∞ and the wall temperature is T_w. The flow along the vertical flat plate contains a species A slightly in the fluid B, the concentration at the plate surface is C_w and the solubility of A in B far away the plate is C_∞..

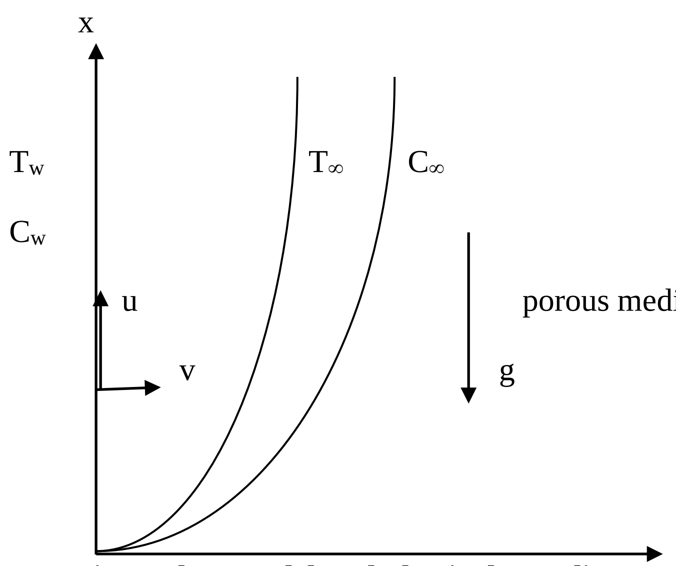

Fig.1. Flow model and physical coordinate system

Several assumptions are applied throughout the paper: (i) the fluid and porous medium are in local thermodynamic equilibrium (ii) the flow is laminar, steady state and two dimensional (iii) the porous medium is isotropic and homogeneous (iv) the properties of the fluid and porous medium are constant (v) the Boussinesq approximation is valid and the boundary layer approximation is applicable (vi) the concentration of dissolved A is small enough. With these assumptions, the governing equations describing the conservation of continuity, momentum, energy and concentration can be written as follows:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$u=\frac{gK}{\upsilon}\left[\beta_T\left(T-T_\infty\right)+\beta_C\left(C-C_\infty\right)\right] \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{C_S C_p}\frac{\partial^2 C}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho C_\rho}\left(T-T_\infty\right)-\frac{1}{\rho C_\rho}\frac{\partial q_r}{\partial y} \tag{3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

All quantities are defined in the nomenclature.

The boundary conditions of the problem are

$$\begin{aligned} &y=0; \quad v=0,\ T=T_w, \quad C=C_w \\ &y\to\infty;\ u\to 0,\ T\to T_\infty,\ C\to C_\infty \end{aligned} \tag{5}$$

where T_w, T_∞, C_w and C_∞ have constant values.

Using the Rosseland approximation for radiation[11], radiative heat flux is simplified as :

$$q_r=-\left(\frac{4\sigma^*}{3k_1}\right)\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and k_1 is the mean absorption coefficient. It is assumed that temperature differences within the flow are sufficiently small such that T^4 can be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about the free stream temperature T_∞ and neglecting higher-order terms. This result in the following approximation:

$$T^4\cong T_\infty^4+\left(T-T_\infty\right)4T_\infty^3=4T_\infty^3 T-3T_\infty^4 \tag{7}$$

Using equations (6) & (7) in (3) we get

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{C_S C_p}\frac{\partial^2 C}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho C_\rho}\left(T-T_\infty\right)+\left(\frac{16\sigma^* T_\infty^3}{3\rho c_p k_1}\right)\frac{\partial^2 T}{\partial y^2} \quad (8)$$

Equations (1), (2), (4), (5), (7) are now non-dimensionalized using the following quantities:

$$\psi=\alpha_m Ra_x^{1/2} f(\eta) \quad where \ \eta=\frac{y}{x}Ra_x^{1/2}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \quad \phi=\frac{C-C_\infty}{C_w-C_\infty} \ \&$$

$$Ra_x=\frac{gK\beta(T_w-T_\infty)x}{\upsilon\alpha_m} \quad (9)$$

The stream function ψ is defined in the usual way

$$u=\frac{\partial\psi}{\partial y}, \quad v=\frac{\partial\psi}{\partial x} \quad (10)$$

The governing equations become

$$f^{'}=\theta+N\phi \quad (11)$$

$$(1+R)\theta^{''}+f\theta'+D_f\phi''+Ecf^{''2}+S\theta=0 \quad (12)$$

$$\frac{1}{Le}\phi''+f\phi'+S_r\theta''=0 \quad (13)$$

where Le, D_f, S_r, N , Ec, R and S are Lewis, Dufour, Soret, Sustentation, Eckert, Radiation and Heat Source respectively.

$$Le=\frac{\alpha_m}{D_m}, \quad D_f=\frac{D_m k_T\left(C_w-C_\infty\right)}{C_S C_p\alpha_m\left(T_w-T_\infty\right)},$$

$$S_r=\frac{D_m k_T\left(T_w-T_\infty\right)}{C_s C_p\alpha_m\left(C_w-C_\infty\right)}, \quad N=\frac{\beta_c\left(C_w-C_\infty\right)}{\beta_T\left(T_w-T_\infty\right)},$$

$$R=\frac{16\sigma^* T_\infty^3}{3\rho c_p\alpha_m Ra_x}, \quad S=\frac{Q}{\rho c_p\alpha_m Ra_x},$$

$$Ec=\frac{\mu\alpha_m Ra_x^2}{x^2\left(T_w-T_\infty\right)}$$

We see that N is positive for thermally assisting flows, negative for thermally opposing flows and zero for thermal driven flows. The prime denotes differentiation with respect to η.

The transformed boundary conditions are

$$f(0)=0, \quad \theta(0)=1, \quad \phi(0)=1 \quad as \quad \eta=0$$
$$f'(\infty)\to 0, \quad \theta(\infty)\to 0, \quad \phi(\infty)\to 0 \quad as \quad \eta\to\infty \quad (14)$$

The parameters of engineering interest for the present problem are the local Nusselt number and local Nusselt number and local Sherwood number and is given by

$$\frac{Nu_x}{Ra_x^{1/2}}=-\theta'(0) \qquad \frac{Sh_x}{Ra_x^{1/2}}=-\phi'(0) \quad (15)$$

Mathematical Solution

The set of non-linear differential equations (11)- (13) with the boundary condition (14) have been solved numerically by applying Runge Kutta Gill method along with the shooting technique. In all cases a step size of $\Delta\eta=0.001$was chosen to be satisfactory for a convergence criteria of 10^{-6}. The value of $\eta_\infty$was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$ .The code was run for the model with two different step sizes $\Delta\eta = 0.01$, $\Delta\eta = 0.001$and in each case the results were in good agreement.

Results and Discussion

In order to discuss the problem under consideration, the results of numerical calculations are presented in the form of non- dimensional velocity, temperature and concentration profiles. Numerical computations have been carried out for different values of Lewis number Le, Dufour parameter D_f, Soret parameter S_r, Sustentation parameter N, Eckert number Ec, Radiation parameter R and Heat Source .

The velocity profiles are displayed from Fig.2 to Fig. 5. In Fig. 2 it is seen that as the buoyancy ration N increases the velocity profile also increases. In Fig.3 it is observed that as the Lewis number increases the velocity profile increases. The influence of viscous dissipation parameter Ec is shown in Fig.4 and it states the relationship between the kinetic energy of the flow and the enthalpy. It is observed that the greater viscous dissipative heat causes an increase in the velocity profiles across the boundary layer. In Fig.5 it is observed that the thermal radiation enhances convection flow such that as thermal radiation intensity R increases, flow velocity profile increases within the thermal buoyancy layer very close to the plate. Furthermore, it is noted that an increase in the thermal radiation parameter produces a significant increase in the thermal condition of the fluid and its thermal boundary layer.

The temperature profiles are plotted through Fig. 6 to Fig.10. It is clearly seen in Fig.6 that the temperature distribution in the boundary layer increases with an increase in the soret number from which we conclude that the fluid temperature rises due to greater thermal diffusion. In Fig. 7 it is investigated as value of the dufour parameter increases, the fluid temperature distribution decreases. The temperature profiles decreases with increase of viscous dissipation parameter Ec in Fig. 8.In Fig. 9 it is noted that the effect of radiation is to decrease the rate of energy transport to the fluid, there by decreasing the temperature of the fluid. In Fig. 10 it is observed that there is increase in temperature profile with increase in heat source parameter S and this is due to the fact that heat is generated.

The concentration profiles are plotted from Fig. 11 to Fig. 13. It is noted that the thickness of the concentration boundary layer decreases as the Lewis number (Le) increases in Fig. 11. Also it is noted that in Fig. 12 as the viscous dissipation parameter increases, the concentration of boundary layer decreases. In Fig. 13 it is seen that there is no marginal increase or decrease of concentration profiles as heat source S increases.

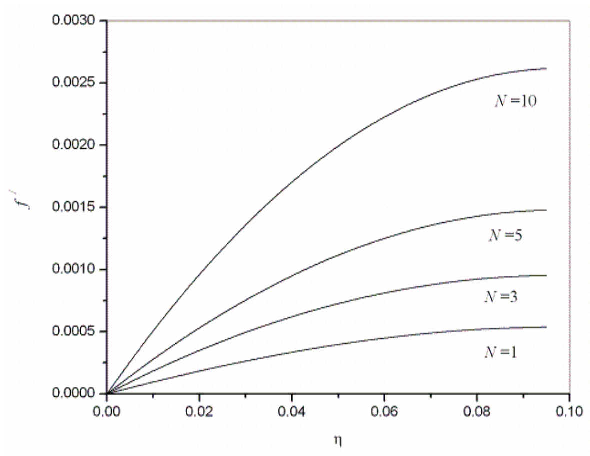

Fig.2 Velocity profile for various values of N with S_r=0.001, D_f=10

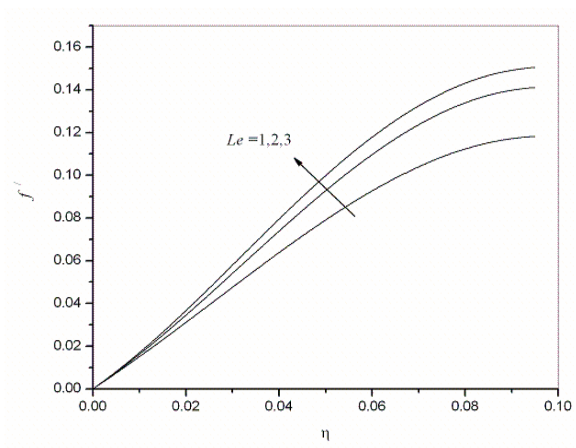

Fig.3 Velocity profiles for various values of Le with S_r=0.001, D_f=10, N =1

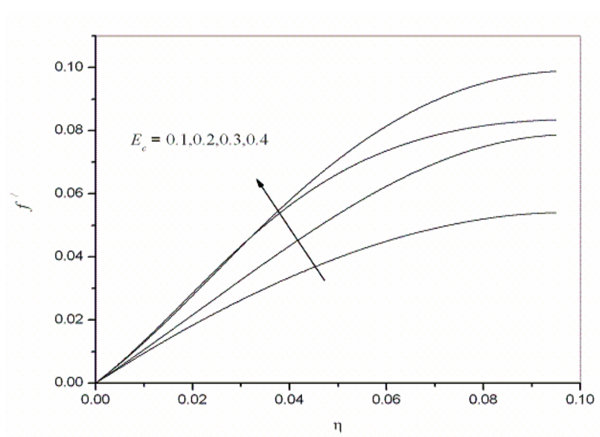

Fig. 4 Velocity profiles for various values of Ec with S_r=0.001, D_f=10, N =1

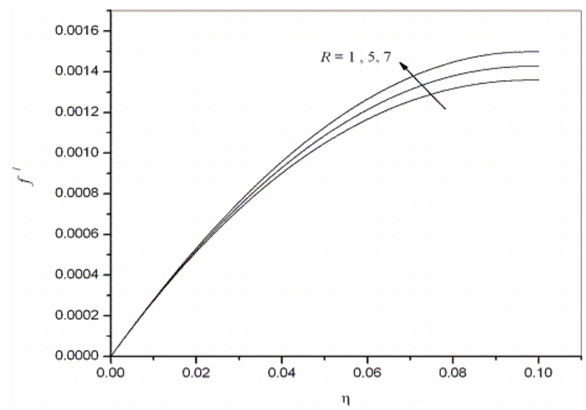

Fig.5 Velocity profiles for various values of R with S_r=0.001, D_f=10, N = 1, Ec=0.5, Le =2

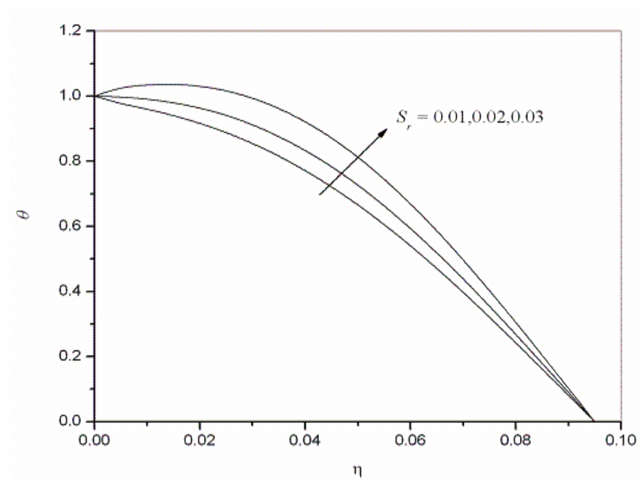

Fig.6 Temperature profiles for various values of soret number with D_f=10, N=1, Le=2, Ec=0.5

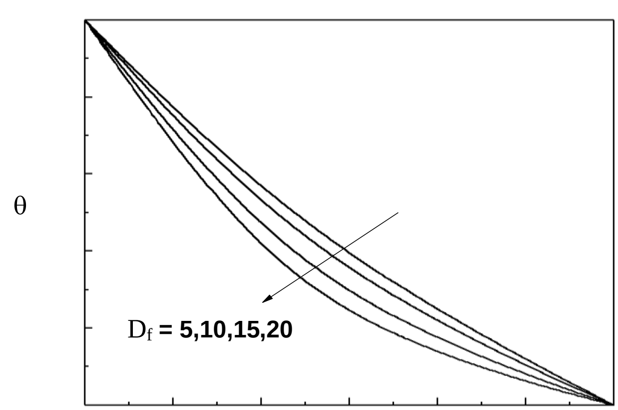

Fig.7. Temperature profiles for various values of dufour number with S_r=0.01, N=1, Le=2, Ec=0.5

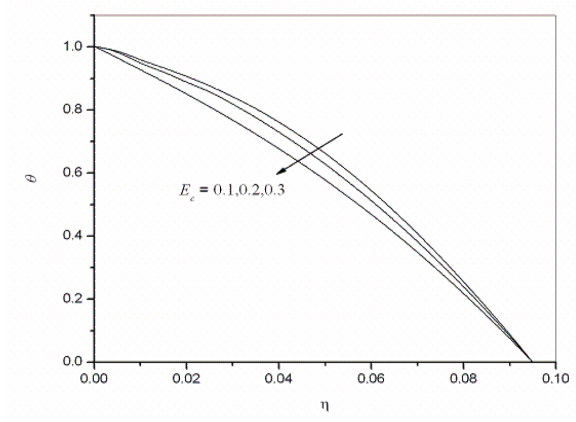

Fig.8 Temperature profiles for various values of Ec with S_r=0.001, D_f=10, N=1, Le=2

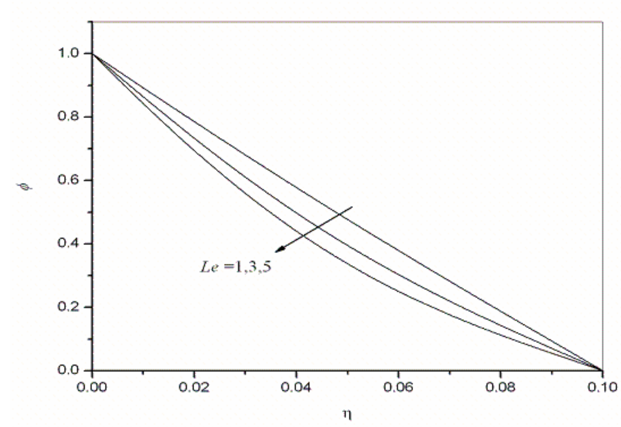

Fig.11 Concentration profiles for various values of Le with S_r=0.001, D_f=10, N=1, Ec=0.5, R=5

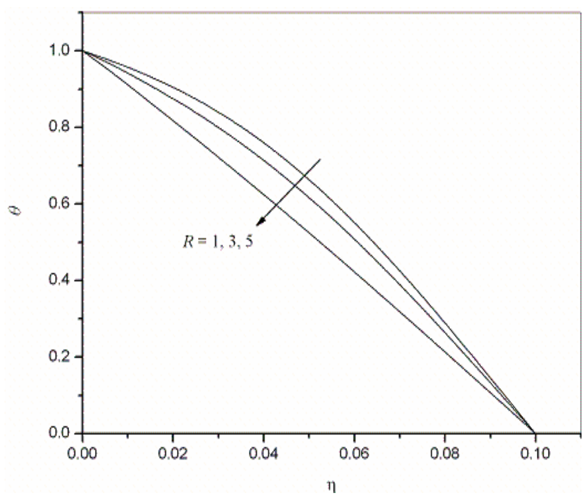

Fig.9 Temperature profiles for various values of R with S_r=0.001, D_f=10, N=1, Ec=0.5, Le=2

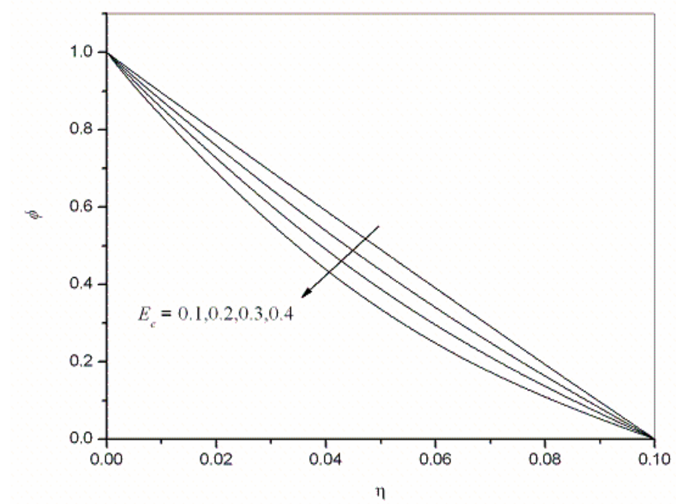

Fig. 12 Concentration profiles for various values of Ec with S_r=0.001, D_f=10, N=1, Le=2, R=5

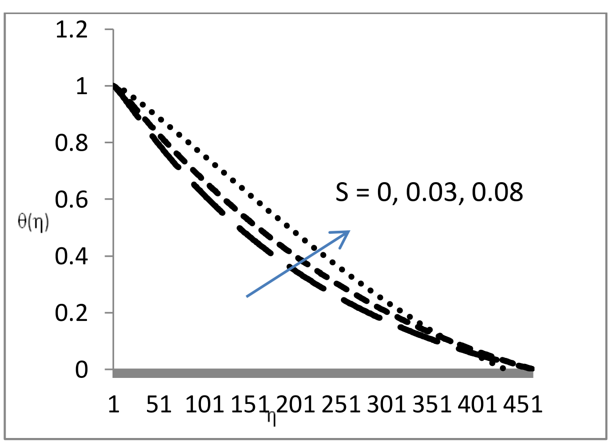

Fig.10 Temperature profiles for various values of S with S_r=0.001, D_f=10, N=1, Ec=0.5, Le=2, R=5

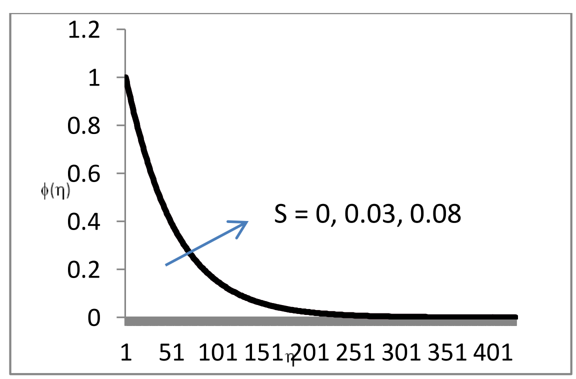

Fig. 13 Concentration profiles for various values of S with S_r=0.001, D_f=10, N=1, Le=2, R=5

Conclusion

This study investigated an approximate analysis on the problem of an steady heat and mass transfer flow of a vertical plate embedded in a darcy porous medium with viscous dissipation under the influence of thermal radiation, heat generation, soret and dufour. Rosseland diffusion approximation has been used to describe and model radiative heat loss on the flow. In the analysis, Runge Kutta Gill method along with the shooting techniques is employed to solve the resulting coupled non-linear partial differential equations. The following conclusions are drawn from the study.

- As the Lewis number increases the velocity profile increases whereas the concentration profile decreases
- The effect of radiation is to decrease the rate of energy transport to the fluid, there by decreasing the temperature of the fluid and the thermal radiation enhances convection flow.
- As the viscous parameter increases the velocity profile increases whereas the temperature and concentration profiles decrease.

Refernces

[1] D A Nield, A Bejan, Convection in porous media, Springer: New York; (2006).

[2] D Ingham, I Pop, Transport phenomena in Porous media I. Oxford Pergamon; (1998).

[3] D Ingham, I Pop, Transport phenomena in Porous media II. Oxford Pergamon; (2002).

[4]A K Acharya, G C Dash, S R Mishra, Free convective fluctuating MHD flow through porous media past a vertical porous plate with variable temperature and heat source. Physics Research International; Article ID 587367, (2014).

[5] P M Kishore, V Rajesh, S Vijay Kumar Varma, The effect of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface condition. Theoret. Appl. Math. 39(2): 99-125, (2012).

[6]S R Mishra, G C Dash, M Acharya, Free convective flow of visco-elastic fluid in a vertical channel with dufour effect, World Applied Sciences Journal, 28(9); 1275-1280, (2013).

[7] P Mohan Krishna, G Vidyasagar, N Sandeep, V Sugunamma, Soret and dufour effects on MHD free convective flow over a permeable stretching surface with chemical reaction and heat source, International Journal of Scientific & Engineering Research, 6(9) 145-147 , (2015).

[8] G Sreedhar Sarma, K Govardhan, Thermo-diffusion and diffusion-thermo effects on free convection heat and mass transfer from vertical surface in a porous medium with viscous dissipation in the presence of thermal radiation, Archives of Current Research International, 3(10; 1-11, (2016).

[9] D Srinivas Reddy, G Sreedhar Sarma, K Govardhan, Effect of viscous dissipation soret and dufour effect on free convection heat and mass transfer from vertical surface in a porous medium, Procedia Materials Science, 10, 563-571, (2015).

[10] R S Tripathy, G C Dash, S R Mishra, S Baag, Chemical reaction effect on MHD free convective surface over a moving

vertical plane through porous medium, Alexandria Engineering Journal, 54; 673-679, (2015).

[11] C H Chen, MHD mixed convection of a power-law fluid past a stretching surface in the presence of thermal radiation and internal heat generation/absorption, International Journal of Nonlinear Mechanics, 44(6), 296-303, (2008).

[12] M. Dakshinamoorthy, P. Geetha and M. B. K. Moorthy, Boundary Layer Flow and Heat Transfer over a Continuous Surface in the Presence of Hydromagnetic Field, International Journal of Mathematical Analysis, 8(38), 1859 – 1872, (2014).

[13] P.Geetha, Soret and Dufour Effects on Steady Free Convection and Mass Transfer Flow Past a Semi-infinite Flat Plate with Viscous Dissipation, Asian Journal of

Research in Social Sciences and Humanities 7(1), 308-314, (2017).

[14] P. Geetha, Dufour and Soret Effects on Thermal Stratification and Non-Newtonian Fluid Parameter on Convective Heat and Mass Transfer along a Horizontal Plate in a Porous Medium, International Journal of Latest Technology in Engineering, Management & Applied Science ,V(IX), 2278-2540, (2016).